Exercise 89

A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

(a) Find the velocity and acceleration functions.

(b) When is the particle moving upward and when is it moving downward?

(c) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.

(d) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 3$.

(e) When is the particle speeding up? When is it slowing down?

Solution

Part (a)

The velocity is the derivative of the position function.

$$\begin{aligned} v(t) &= \frac{dy}{dt} \\ &= \frac{d}{dt}(t^3 - 12t + 3) \\ &= 3t^2 - 12 \end{aligned}$$

The acceleration is the derivative of the velocity function.

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt}(3t^2 - 12) \\ &= 6t \end{aligned}$$

Part (b)

The particle is moving upward when

$$v(t) > 0$$

$$3t^2 - 12 > 0$$

$$3(t^2 - 4) > 0$$

$$t < -2 \quad \text{or} \quad t > 2,$$

but since $t \geq 0$, $t > 2$.

The particle is moving downward when

$$v(t) < 0$$

$$3t^2 - 12 < 0$$

$$3(t^2 - 4) < 0$$

$$-2 < t < 2,$$

but since $t \geq 0$, $0 \leq t < 2$.

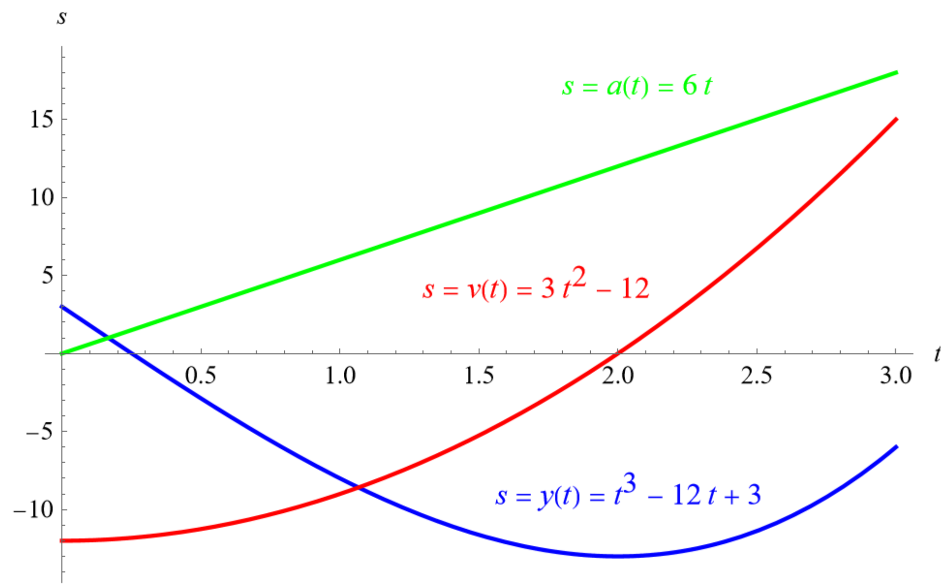
Part (c)

Add up the distances the particle travels when it's moving up and moving down separately. It was found in part (b) that the particle moves down on $0 \leq t < 2$ and moves up on $t > 2$.

$$\begin{aligned}
s &= \int_0^3 |v(t)|\, dt \\
&= \int_0^2 [-v(t)]\, dt + \int_2^3 [v(t)]\, dt \\
&= -\int_0^2 v(t)\, dt + \int_2^3 v(t)\, dt \\
&= -[y(2) - y(0)] + [y(3) - y(2)] \\
&= -y(2) + y(0) + y(3) - y(2) \\
&= y(3) - 2y(2) + y(0) \\
&= [(3)^3 - 12(3) + 3] - 2[(2)^3 - 12(2) + 3] + [(0)^3 - 12(0) + 3] \\
&= (-6) - 2(-13) + (3) \\
&= 23
\end{aligned}$$

Part (d)

Below is a graph of the position, velocity, and acceleration functions on $0 \leq t \leq 3$.



Part (e)

The particle is speeding up if either both $v(t)$ and $a(t)$ are positive or both $v(t)$ and $a(t)$ are negative. This condition is satisfied when

$$t > 2.$$

The particle is slowing down if the velocity and acceleration have opposite signs. This condition is satisfied when

$$0 < t < 2.$$